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Direct Blood Pressure Measurement—Dynamic Response Requirements

*Reed M. Gardner, Ph.D.**

IN 1903 Otto Frank established the initial criteria for recording pressure waveforms with adequate fidelity.[1] Several other groups pursued these investigations between 1903 and 1949; in 1949, Hansen[2] published one of the classic studies on dynamic response and its theoretical and practical expressions. Since 1949, several studies (Wood,[3] Fry,[4] Yanof *et al.*,[5] Latimer and Latimer,[6,7] and Shapiro and Krovetz,[8]) have been conducted using animals and humans. These investigators have suggested different dynamic response criteria. This paper establishes requirements for adequate dynamic response, suggests methods for testing the fidelity of catheter-tubing-transducer systems in the clinical setting and characterizes a new device (Accudynamic®) which can be used to optimize dynamic response of direct blood pressure measuring systems.

Catheter-transducer systems used in intensive care can usually be characterized by an "underdamped, second-order dynamic system," which is analogous to a bouncing tennis ball. When a tennis ball is dropped on a hard, flat floor it bounces several times and comes to rest on the floor. With each successive bounce it does not rise as high as on the previous bounce. Each bounce has a characteristic frequency, and the time it takes the ball to come to rest is related to the damping coefficient.

A second-order system can be characterized by three mechanical parameters: elasticity, mass, and friction. These same parameters apply to a catheter-transducer system and are defined as follows: 1) elasticity—the stiffness of the system, normally caused by the flexibility of the transducer diaphragm; this can be altered by air bubbles, compliant tubing, or other elastic elements in the system; 2) mass—the fluid mass moving in the system, usually in the catheter and interconnecting tubing; and 3) friction—friction in the catheter and tubing as the fluid in them moves with each pulsatile change in blood pressure. Theoretically,[2] the catheter-tubing-transducer system is a distributed system. However, in the clinical setting it can be approximated by a simple second-order system.[4,8,9,10] Experimental results for frequencies up to 50 Hz, show that most catheter-tubing-transducer systems follow a second-order model.[4] We have observed that several types of catheter-tubing-transducer systems respond like simple underdamped dynamic systems. That is, they act like the bouncing ball by oscillating for a time and then coming to rest. Therefore, the discussion which follows will be concerned with an underdamped second-order system.

With the underdamped second-order system, the elasticity, mass and friction determine two measureable parameters: the natural frequency (f_n), and the damping coefficient (ζ). Both are measurable parameters in the mechanical, as well as in the catheter-tubing-transducer systems. The natural frequency refers to how rapidly the system oscillates, and the damping coefficient refers to how quickly the system comes to rest. Detailed discussions of the requirements for systems and their characterizations are found in two excellent books by Geddes[9] and McDonald.[10]

Several techniques which characterize the dynamic response of catheter-transducer systems have been proposed. These techniques are generally separated into two categories: frequency bandwidth requirement, and specification of natural frequency and damping coefficient. The frequency bandwidth requirement is usually specified in terms such as "flat to 16 Hz" or "flat to the 6th or 10th harmonic" of the pressure signal.[9,10] The specification of a natural frequency and damping coefficient of the system is the better method, since these two parameters

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* Professor, Department of Medical Biophysics and Computing.

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Address reprint requests to Dr. Gardner.

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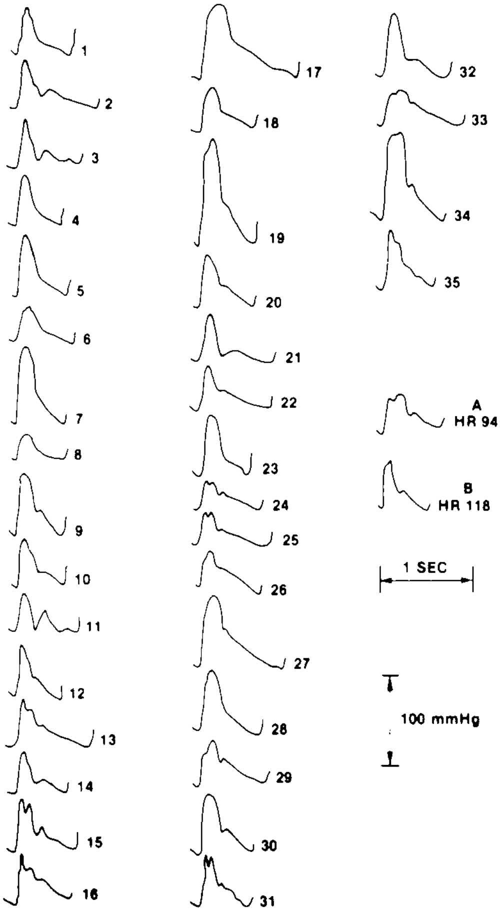


FIG. 1. Shown above are arterial pressure waveforms recorded from 37 different patients. The catheter-transducer system used to record them had a damping coefficient of approximately 0.2 and natural frequency > 25 Hz. The wide variety of wave shapes is typical of what is seen in the clinical environment and shows that waveform inspection alone is inadequate for assessing dynamic response characteristics. Waveform A is a typical arterial or pulmonary artery pulse waveform. Waveform B is for a faster heart rate and has a rapid pressure upstroke—this waveform is one of the most demanding and requires the greatest system dynamic response.

completely characterize a second-order system. It should be possible to compare the two methods of characterization of dynamic response, however, because of non-uniform frequency content of the pressure signal and characteristics of the catheter-transducer system, this is seldom done. As far as this author has been able to determine, there has been little quantitative comparison of the clinical needs with the theoretical specification methods.

Dynamic response requirements of blood pressure monitoring systems can be divided into three areas of application: 1) the research area, requiring very wide bandwidth, perhaps going into the audio frequency range, *e.g.*, certain types of measurement technologies, such as differential pressure measurements: 2) the catheterization laboratory area, where there may be different requirements than those for patient monitoring, *e.g.*, where the measurement of left ventricular dP/dt sometimes is used as an assessment of left ventricular function; and 3) the clinical area where patients are located in intensive care units and surgical operating suites. This study addresses the third area. For clinical use, underdamped catheter-transducer systems result in two major errors in blood pressure reading: overestimation of systolic pressure frequency by as much as 15–30 torr, and amplification of artifact in the measurement system such as catheter whip in the pulmonary artery. Since most electronic monitoring systems look only at peak and valley pressures for systole and diastole, both of the above factors can cause major clinically relevant errors.

Methods

In order to characterize and establish criteria for high fidelity recording in the clinical setting it was necessary to record and characterize several pressure waveforms. During a six-month interval, waveforms from 37 patients were recorded on magnetic tape (see fig. 1), and the resulting data showed that there were large variations in pulse pressure, heart rate, and wave shape for arterial waveforms. It is interesting to note that although all the waveforms were recorded using systems which had good fidelity (*i.e.*, natural frequencies above 25 Hz and damping coefficients near 0.25), there were still great variations in the wave shapes. There were several waveforms which did not have discernible dicrotic notches (*e.g.*, numbers 4, 5, 6, 7, 17, 23, and 28). The thought may occur to the critical observer that the recording system may have been "overdamped" resulting in the waveform distortion; however such was not the case. Other waveforms (*e.g.*, numbers 11, 13, 15, 16, and 31) seemed to have wave shapes recorded with "underdamped" systems. However, in all cases, the fidelity was adequate and the waveforms recorded were actual physiological variations. There were three important observations made from these waveforms: 1) there was a large physiological variation between patients; 2) there were differing dynamic response requirements for each patient waveform; and 3) simple visualization of the waveform does not give sufficient information to determine adequacy of dynamic response. Two waveforms were chosen. Waveform A, from a patient with a heart rate of 94 beats/min, was selected because it was typical of arterial waveforms found in our patient population. Waveform B was chosen because it required the greatest dynamic re-

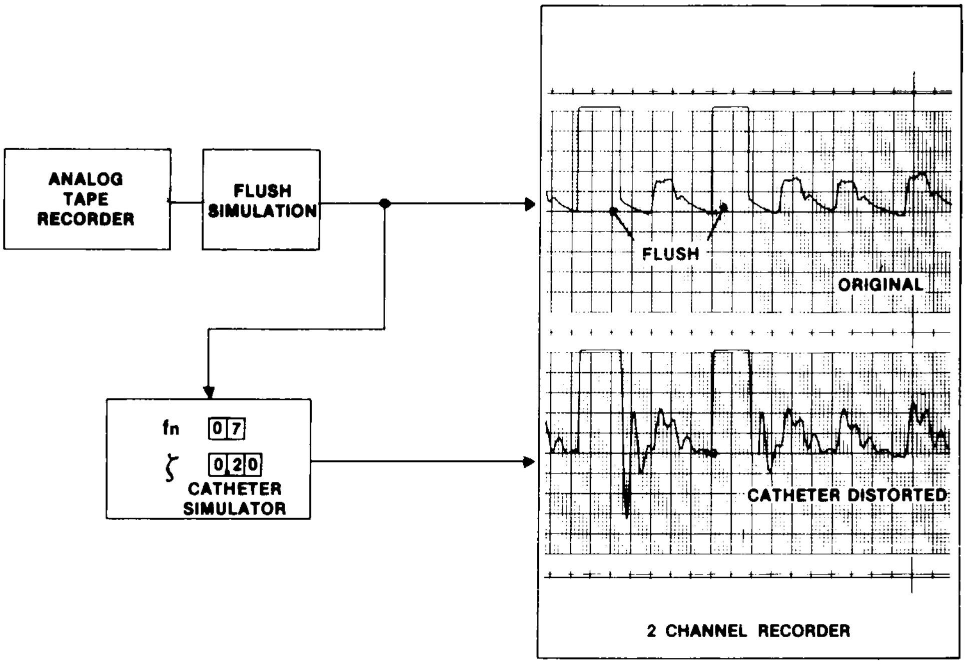


FIG. 2. Outlined is the system used to assess pressure wave dynamic response requirements. The upper recorder panel shows the original waveform A with two flushes superimposed while the lower panel shows the waveform after it has been transmitted through a simulated catheter system. Note the "ringing" in the flush signal and the dramatic change in wave shape because of low natural frequency (7 Hz) and small damping coefficient (0.20).

sponse of the 37 patient waveforms. This was due to the rapidly rising systolic upstroke, or dP/dt on the systolic ejection portion of waveform B.

Using the system outlined in figure 2, the recorded waveforms were processed through a catheter-transducer simulator. The two squarewave signals were added to the recorded waveforms to simulate a "fast flush" signal. The fast flush signal can be generated in monitoring systems by opening the system to a high pressure flush bag using an Intraflo® continuous flush element or a quick turn of a stopcock. The flushes were inserted to show the effect a squarewave transient had on the simulated catheter. The upper graph in figure 2 shows the original waveform and the lower shows the distortion caused by the simulated catheter. The catheter simulator was an electronic analog computer which permitted independent adjustment of natural frequency and damping coefficient. The simulator had a natural frequency ranging from 1–45 Hz, in 1 Hz increments, and a damping coefficient ranging from 0.05–1.5, in 0.05 increments. Figure 2 shows a simulated system which has a natural frequency of 7 Hz and a damping coefficient of 0.2. The analog catheter simulator was used to process the original waveforms in order to determine what effects were caused by changing the natural frequency and damping coefficient.

Figure 3 outlines the test methods used to assess the characteristics of catheter-transducer systems. A Biotek Model 601® blood pressure simulator, connected to a sweep frequency generator, was used to drive a reference transducer and the actual (not simulated as in fig. 2) catheter tubing configuration which was being tested. The results shown in figure 3 are for a system with a damping coefficient of 0.23 (underdamped) and a natural frequency of 20.8 Hz. Several catheter-transducer systems used clinically (see table 1) were tested using this system. Individual systems were set up and carefully filled with normal saline to eliminate air bubbles, then tested multiple times. Results from the test with the highest natural frequency are reported. The characteristics of such systems are determined by the transducer used, the type dome used on the transducer (disposable *vs.* nondisposable), interconnecting tubing material diameter and length, the catheter type and, most importantly, the effect of small air bubbles trapped in the system. Therefore, even with the same configuration it is not unusual to have different characteristics (see *e.g.*, numbers 6, 8, 9 and numbers 10, 11 of table 1). Flush testing of these same systems resulted in nearly identical characteristics.

To minimize the pressure errors caused by the resonant peak shown in figure 3 or the related ringing shown in figure 2, we explored several alternative solutions. Electronic compensation, both analog[11,12] and digital,[13] have been proposed. None of these have been developed into commercial products so we investigated mechanical compensation methods.[6,14,15] After careful testing of the series resistive element suggested by Hansen[2] and the parallel "impedance

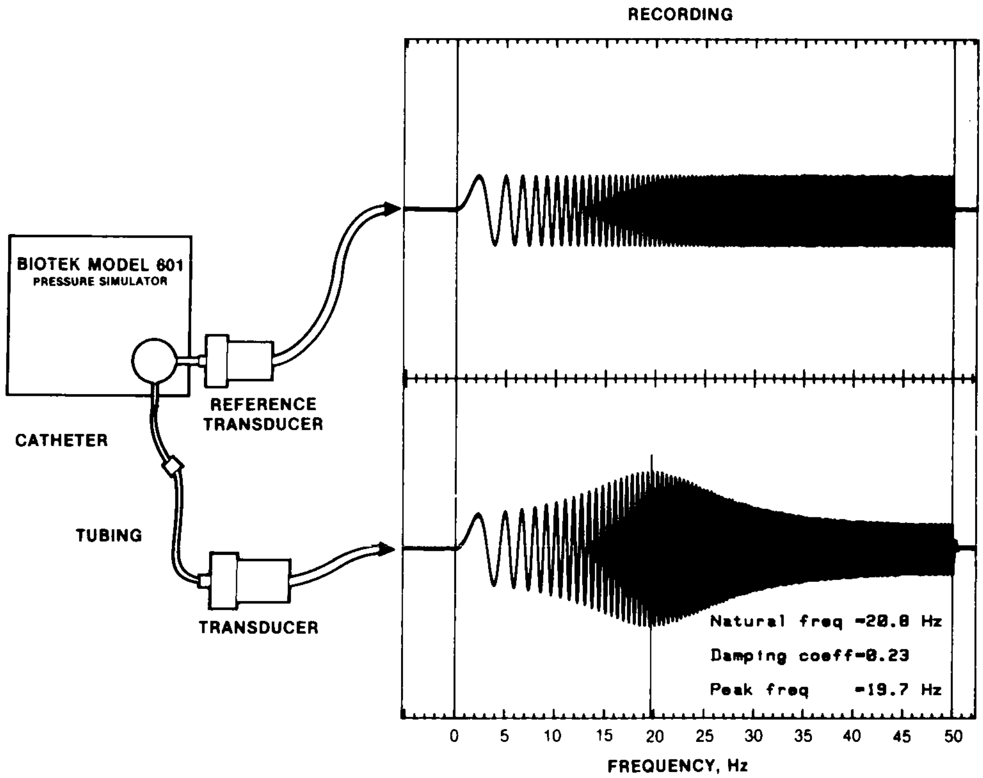


FIG. 3. Shown is a block diagram of the system used to sinusoidally test catheter-transducer systems. From these data one can determine the natural frequency (20.8 Hz) and damping coefficient (0.23). A monitoring system with these characteristics would cause a small overshoot and thus overestimation of systolic pressure. See figures 5 and 6 which graphically illustrate the distortion.

TABLE 1. Catheter-tubing-transducer System Characteristics*

No.	Description	Natural Frequency (Hz) (f_n)	Damping Coefficient (ζ)
1	5 Fr Two Lumen Pulmonary Artery HP Transducer Dyne Diaphragm Dome	9.5	0.32
2	5 Fr Two Lumen Pulmonary Artery HP Transducer HP Diaphragm Dome	10.0	0.30
3	4 Fr Two Lumen Pulmonary Artery (47 cm) Bell & Howell Transducer and Diaphragm Dome	12.0	0.30
4	6 Fr Two Lumen Pulmonary Artery HP Transducer No Diaphragm Dome	13.0	0.15
5	5 Fr Two Lumen Pulmonary Artery HP Transducer No Diaphragm Dome	14.0	0.25
6	7 Fr Four Lumen Thermodilution Pulmonary Artery (#1)	14.5	0.20
7	CAP† 18 GA with 24" pressure tubing	15.0	0.72
8	7 Fr Pulmonary Artery (#2) (See #6)	15.5	0.20
9	7 Fr Pulmonary Artery (#3) (See #6)	14.0	0.32
10	Vinca‡ + 84" Pressure Tube (#1)	16.0	0.10
11	Vinca + 84" Pressure Tube (#2) (See #10)	16.0	0.20
12	CAP 18 GA Direct	20.0	0.30
13	48" Pressure Tubing Direct	24.0	0.28
14	7 Fr Pulmonary Artery	25.0	0.15
15	Vinca + 24" PVC Pressure Tubing	38.0	0.10
16	Vinca + 48" PVC Pressure Tubing	45.0	0.13
17	Vinca + 24" Polyethylene Pressure Tubing	48.0	0.14

* Unless otherwise specified runs made with Bentley Model 800 Transducer without a diaphragm dome.

† CAP = 18 Gage CAP Catheter-Sorenson.

‡ Vinca = 2" 18 Gage over the needle arterial catheter.

matching" system proposed by Van der Tweel,[14] Latimer and Latimer[6], and Vierhout,† it was found that the parallel impedance matching was easiest to implement and use in the clinical setting.

Figures 4*A* and 4*B* show the device (Accudynamic®) designed and constructed to meet these requirements.‡ The device consists of a 0.1-ml sealed air bubble and an adjustable needle valve resistor used to optimize the impedance matching.

Results

Figure 5 shows a plot of some of the simulated catheter distortion data obtained from waveform A. At a natural frequency of 7 Hz and a damping coefficient of 0.5, the catheter distorted waveform was almost identical to the original waveform. Criteria for selecting distortion were based on visual comparisons of the waveforms, since waveform subtraction did not account for phase delays caused by the catheter-transducer system. There was considerable oscillation in the waveform for the same natural frequency, and a damping coefficient of only 0.2. When the damping coefficient was 1.0 (critically

† Vierhout RR: The response of catheter-manometer systems used for pressure recording. Ph.D. Thesis, University of Nijmegen, Holland, 1966.

‡ Accudynamic®, Sorenson Research Company, Salt Lake City, Utah.

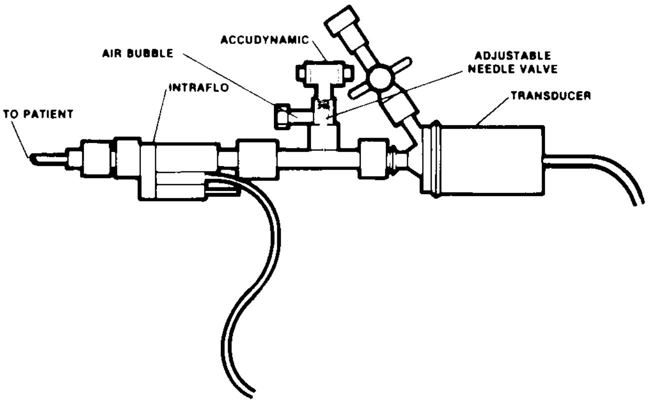


FIG. 4A. The damping device (Accudynamic®) is shown here in a typical location. It could also be placed on the side port of the transducer or between the Intraflo® and the patient connection. Also illustrated are the 0.1 ml sealed air bubble and the adjustable needle valve (resistor) of the device.

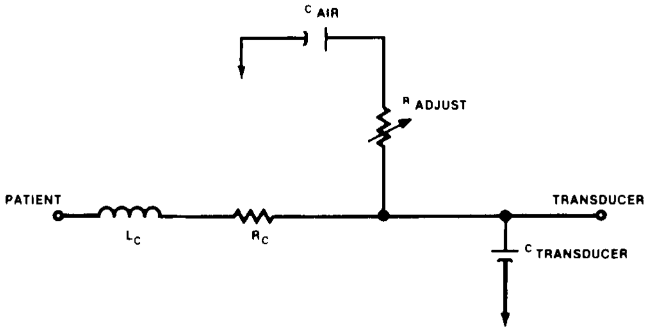
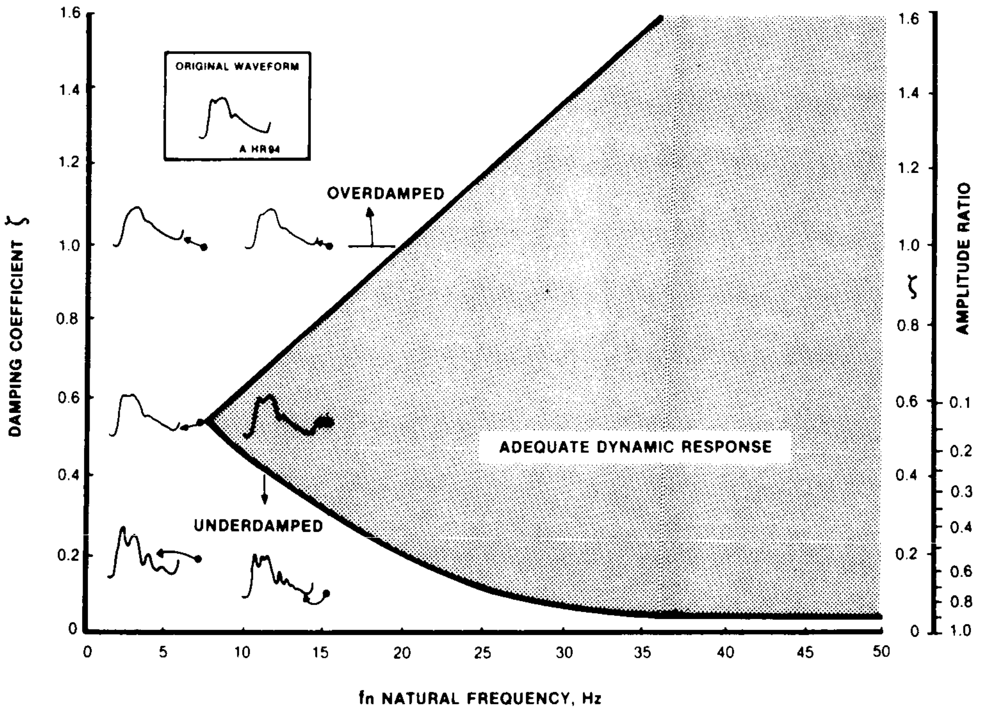


FIG. 4B. A schematic diagram of the Accudynamic® with needle valve (R_{ADJUST}), and C_{Air} a fixed compliance of 0.1 ml. By turning the needle valve, R_{ADJUST} is used to optimize the damping coefficient without decreasing the natural frequency.

damped), the waveform appeared to be "damped" or smoothed out and there was a loss of fine detail such as the dicrotic notch. Figure 5 also shows waveforms with damping coefficients of 0.1, 0.5, and 1.0, at a natural frequency of 15 Hz. With a damping coefficient of 0.1 (underdamped) there was "overshoot" in the waveform, as well as oscillation or ringing. A damping coefficient of 1.0 caused some reduction of the dicrotic notch, and "smoothing" of the waveform, because high frequency components were lost. By utilizing the 0.05 steps, we adjusted the damping coefficient until there was no discernible difference between the original waveform and the catheter-distorted waveform. The stippled area of figure 5 represents a range of natural frequency and damping coefficient which will not distort the waveform from patient A. If the natural frequency of the system is 7 Hz, a damping coefficient of 0.5 is required; whereas, if the natural frequency of the system is 20 Hz, the damping coefficient can range from 0.2 to 1.0 and still accurately reproduce the waveform. Therefore, a major requirement of any catheter-transducer recording system is that it have a high natural frequency to allow for the largest possible latitude in damping coefficient.

Waveform B, processed in a manner similar to waveform A, is shown in figure 6. Note that the faster

FIG. 5. For patient "A" this plot shows the ranges of damping coefficients and natural frequencies which do not distort the pressure waveform (stippled area). For the underdamped region (lower left) the pressure waveform has overshoot (increase in systolic pressure) and "ringing" while for the overdamped region (upper area) there is loss of fine detail in the waveform, as well as a decrease in systolic pressure. For the waveforms shown in this figure, in one there is a maximum overestimation of systolic pressure of 14 torr and in another an underestimate of diastolic pressure of 2 torr. Scale on the right allows conversion from amplitude ratio to damping coefficient (see figure 8 and Appendix).



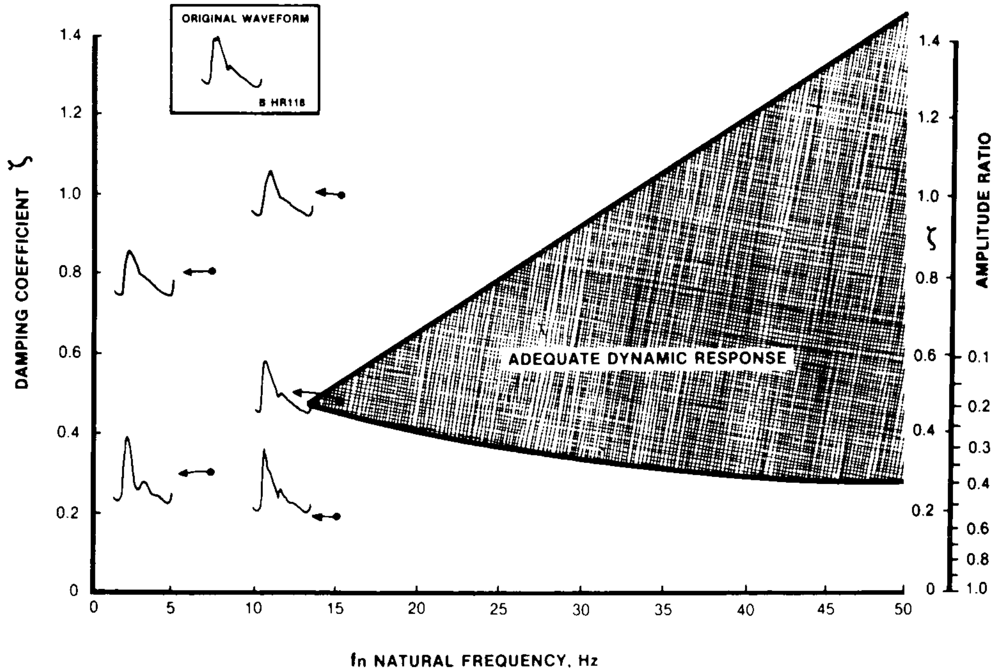


FIG. 6. A plot similar to figure 5 but for patient "B". Note that the natural frequency must be considerably higher (above 13 Hz) and the range of damping coefficient for adequate dynamic response is much more restricted. This results from the rapid pressure rise during systole and the rapid heart rate. For the waveforms shown there is a maximum error of 15 torr overestimate in systolic and 3 torr underestimate in diastolic pressure.

heart rate (HR = 118), in addition to the rapid upstroke of waveform B, requires that the natural frequency be >13 Hz and that there be a more tightly controlled range of damping coefficients. Figures 5 and 6 give the operating band within which catheter-transducer systems should operate.

Figure 7 shows a plot of characteristics of several catheter systems in current use (taken from table 1). The important thing to note from figure 7 is that most monitoring systems had a relatively low natural frequency and were underdamped. Many of the systems would adequately reproduce the more typical

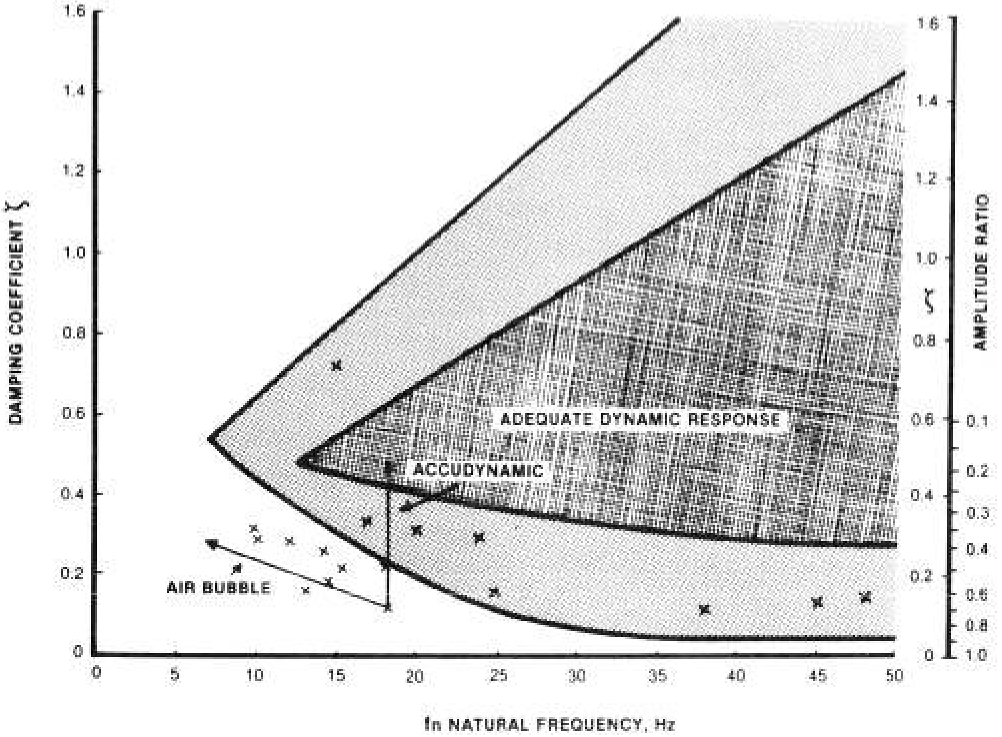


FIG. 7. The ×s shown are for the monitoring systems tabulated in table 1. These system characteristics are overlaid on the dynamic response requirements of patient "A" (fig. 5) and patient "B" (fig. 6). The space between the two "wedges" is the band where most pressure waveforms shown in figure 1 will fall. Thus the patient "A" wedge (fig. 5) is a minimum requirement while the patient "B" wedge (fig. 6) is a maximum requirement for recording accurate arterial pressure waveforms.

waveform (waveform A) but none would adequately record the fast waveform (waveform B). Therefore, the compensation methods mentioned earlier were used to restore the pressure waveforms to an "adequate dynamic response region" (fig. 7).

Discussion

It is well known that adding a small air bubble to a monitoring system, such as in the transducer dome, increases the damping coefficient. However, the introduction of an air bubble also decreases the natural frequency, thereby moving the operating point of the catheter-transducer system upward and to the left (see fig. 7) when ideally it should be upward and to the right. The device developed, however, allows adjustment of the damping coefficient without decreasing natural frequency (see fig. 7).

As the methods for testing dynamic response of pressure monitoring systems were developed, it became apparent that sinusoidal testing and simulator testing were not adequate in the clinical setting. Several investigators[2,3,9,10,15,16] proposed "squarewave" testing. We suggested using the Intraflo® fast flush method to test the dynamics of the system.[17] With underdamped systems, which will oscillate near their natural frequency, the ability to excite the system with a rapid closure of the fast flush valve allowed measurement of natural frequency and of the damping coefficient.[15,17,18]

Figure 8 shows a recording from a flush and characteristic ringing which illustrates the measurement and computational technique. For the range of damping coefficients where the flush method was practical (damping coefficient 0.01–0.4), the natural frequency was within 10 per cent of the measured frequency. Therefore, the natural frequency could be determined by measuring the period (distance) of one cycle and applying the following equation:

$$f_n \approx \frac{\text{Paper speed mm/sec}}{\text{One cycle measured in mm}}\ \text{Hz}$$

For example, in figure 8, the paper speed was 25 mm/sec and the distance for one cycle was 1.7 mm; therefore the natural frequency was 15 Hz. Faster paper speed, or measuring multiple cycles, made measurement of natural frequency more accurate.

The damping coefficient determination required two measurements and a graphical solution (see Appendix). Amplitude measurement (mm) from the ringing curve can be made with *any* two successive peaks [*e.g.,* amplitude (A1) and amplitude (A2) of fig. 8]. Amplitude ratios were taken (dividing the smaller number by the larger number, *e.g.*, A2/A1 = 17/24 = 0.71). The ratio (0.71) was plotted on the right hand scale of figure 5 or determined from the chart or equation in the appendix and gave a damping coefficient of 0.11.

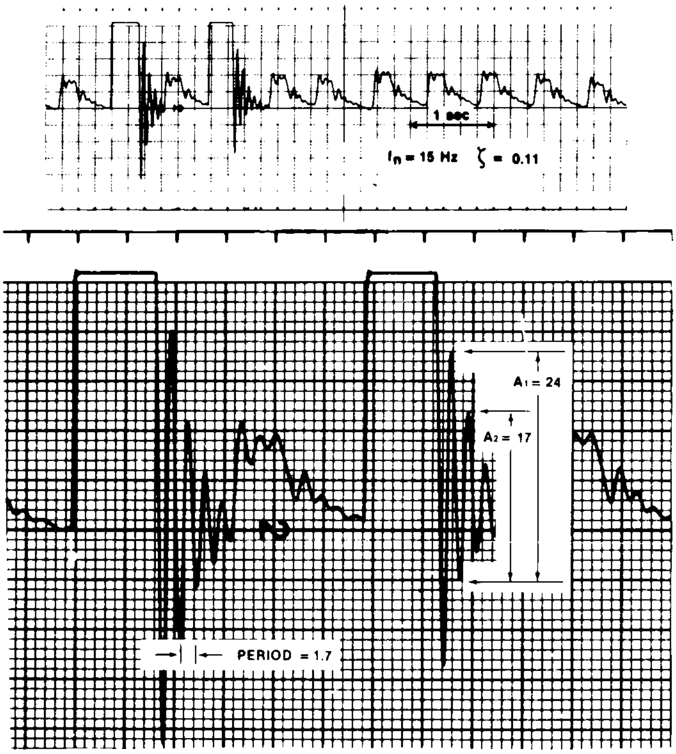


FIG. 8. The upper panel (8*A*) shows an arterial pulse waveform with two flushes. The natural frequency and damping coefficient can be determined from either flush. The lower panel (8*B*) shows the flush segment enlarged and marked to illustrate the method. The natural frequency of the system is estimated by taking the period of one cycle (PERIOD), in this case 1.7 mm, and dividing this into the paper speed 25 mm/s; $f_n = 25/1.7 \approx 15$ Hz. The natural frequency can be more accurately determined by measuring multiple cycles. The damping coefficient is determined by taking the ratio of successive peaks of the oscillations, in this case $A_2/A_1 = 17/24 = 0.71$, then by using the equation or the scale on figure 5, 6 or 7, the damping coefficient ζ can be determined, in this case, ζ = 0.11.

The flush method is superior to other techniques for characterizing catheter-transducer systems because: it can be used to test the entire system, from catheter tip to the waveform recording system; it can be used in a clinical setting without attaching additional devices; and the Intraflo® continuous flush system, with its fast flush valve, is already in place in most of the clinical settings.

In the clinical setting the pulsating pressure waveform will be superimposed on the flush results. Therefore, two or three flushes must be performed to facilitate measurement of the natural frequency and

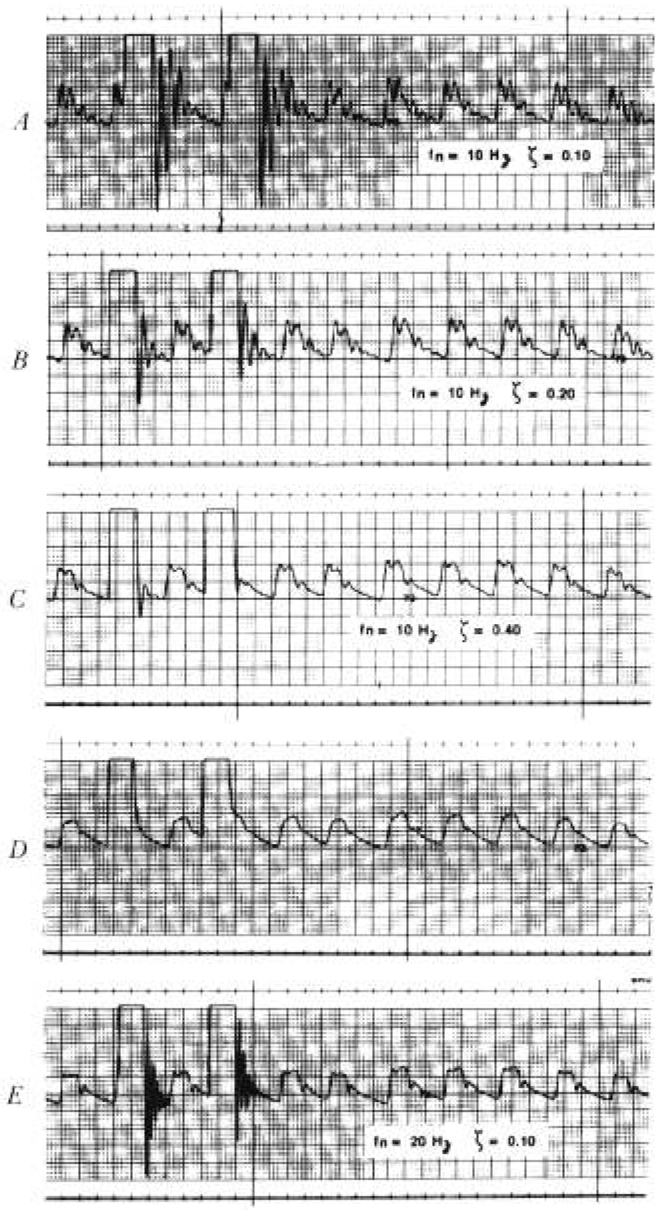


FIG. 9. Five pressure tracings showing method of optimizing damping using the flush technique and Accudynamic®. Each tracing has two flushes. *A*. Patient waveform with two flushes which shows a low natural frequency 10 Hz and a low damping coefficient 0.1. Note overshoot and ringing in the patient waveform. *B*. Damping increased to 0.2 by adjusting the needle valve. Natural frequency the same. Ringing after flushing decays more quickly. *C*. Near optimum damping (0.4). Note that there is little ringing on the flush signals. *D*. Over adjustment of damping on an overdamped system. From the flush the damping coefficient and natural frequency are not easily determined, however the flush clearly indicates overdamping because there is no ringing. *E*. Shows a waveform where the natural frequency is increased to 20 Hz by shortening the interconnecting tubing length but with a damping coefficient of 0.1. Note that the patient waveform is much less distorted than in Figure 9*A*. This figure also illustrates the importance of timing of the flushes to measure the damping. Tracing 9A has two flushes which occur and are distorted by the systolic pulsation of the aortic pressure signal. On the other hand in 9E the flushes were made during the more desirable time of the diastolic runoff when there is less artifact added to the flush signal.

damping coefficient as illustrated in figure 9. Figure 9*A* shows the flush superimposed on the systolic upstroke which gives signals hard to interpret while figure 9*E* shows near optimum placement of the flushes during diastole. The technique is usually only accurate to ±0.03 units of the damping coefficient.

It is necessary to identify and specify both the natural frequency and the damping coefficient (see fig. 5 and 6) for clinically applied pressure monitoring systems. The pressure monitoring systems with the best dynamic response are those which have a high natural frequency (determined by the flush method). A system with a natural frequency < 10 Hz will be marginal even with the Accudynamic® compensation. Suggestions as to how to minimize factors known to lead to monitoring systems with low natural frequencies are listed below. Air bubbles—use transparent tubing and fluid pathways so that air bubbles are easily seen and removed. Air bubbles are not easily detected at interconnecting points and between diaphragm domes and metal transducer diaphragms. Complex systems—keep the system simple with the fewest number of components possible. Compliant elements—use only high quality (low compliance) pressure tubing in as short a length as possible. Use low compliance catheters and transducers, use tightly sealing low compliance stopcocks, and eliminate other compliant elements such as injection sites.

If, after the natural frequency is maximized, the monitoring system falls in the range shown in figure 5 or 6, no further action need to be taken to monitor quality pressure signals. (This occurs in about 90 per cent of our systems because we generally attach the transducer directly to the catheter and tape the transducer to the patient.) However, if the monitoring system is underdamped (low damping coefficient) the system can be optimized by the addition of the Accudynamic®. When this device is installed in the monitoring system near the transducer, the natural frequency should not decrease; if it does, the system has been improperly set up and any air which has been introduced should be removed. Once installed, it should be adjusted to the optimum damping coefficient (see fig. 7 and 9). In practice, the optimum waveforms are recorded when a flush results in one undershoot followed by a small overshoot, then settles to the patient's waveform (see figure 9C). In order to determine natural frequency, it may be desirable to turn the Accudynamic® to the "off" position, since frequency is difficult to measure with damping coefficients above 0.4. Because of variability of catheter-transducer system setups and even differences in characteristics with identical setups, it is necessary to test and optimize each individual monitoring system. Testing of dynamic response should be done at least once each shift and anytime the system has been opened to draw blood or change a component. After an initial learning period, nurses and physicians can estimate adequacy by visual observation of the flush waveforms on an oscilloscope.

Conclusions

Optimum dynamic response is required if systolic and diastolic pressures are to be measured accurately. If mean pressure is the only measurement required, then dynamic response characteristics are of little importance. With increasing use of derived variables, such as rate-pressure product, it is important that correct pressures be measured.

A system with inadequate dynamic response, whether underdamped or overdamped, will result in an error in systolic pressure. The systolic pressure from an underdamped system will be overestimated and the systolic pressure from an overdamped system will be underestimated. Diastolic pressure also is affected, but is much more tolerant of dynamic response inadequacies. If invasive pressure monitoring systems are used, with their attendant risk of complication to the patient, great care should be taken to obtain accurate and reliable data. Simply looking at the waveform does not provide sufficient information for one to determine adequacy of dynamic response. This paper outlines requirements for adequate system dynamic response, gives simple methods for clinical testing and presents a device for optimizing dynamic response.

It is recommended that any subsequent reports in which direct systolic blood pressure is a critical measurement, should provide the natural frequency and damping coefficient of the catheter-tubing-transducer system. Also, manufacturers should be encouraged to specify the natural frequency and damping coefficients for their catheter-transducer-tubing systems.

The author acknowledges the help of Blair J. West for designing and constructing the catheter simulator and Gordon Reynolds and Jim Strom for assistance with frequency response testing.

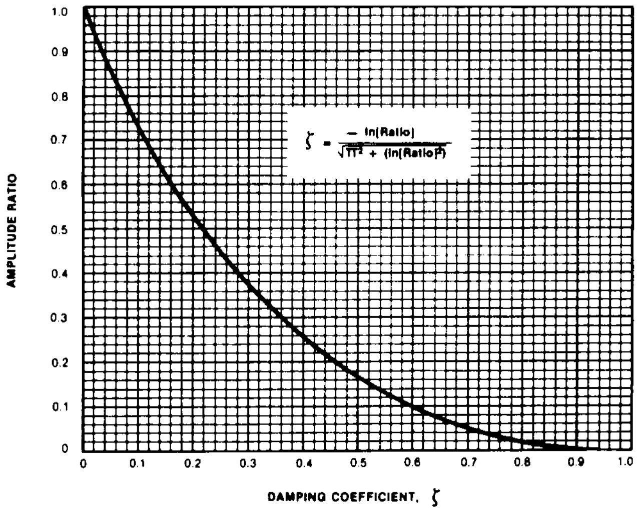


FIG. 10. Graphical solution of equation 3 of the Appendix.

APPENDIX

Equation 1 is the equation which describes the oscillation of a second-order system to a step response.[9,16]

$$P(t) = Po - \frac{Po}{(1-\zeta^2)^{1/2}} e^{-\zeta 2\Pi f_n t} \sin(2\Pi f_n(1-\zeta^2)^{1/2}t) \quad (1)$$

If this equation is solved at three successive peaks, that is where t equals

$$t_1 = \frac{1}{2f_n(1+\zeta^2)^{1/2}} \quad t_2 = \frac{1}{f_n(1-\zeta^2)^{1/2}} \quad t_3 = \frac{3}{2f_n(1-\zeta^2)^{1/2}},$$

then by subtracting the difference and taking the ratios

$$\text{Ratio} = \frac{A_2}{A_1} = \frac{e^{-\zeta\Pi}}{(1-\zeta^2)^{1/2}} \quad (2)$$

By solving this equation the damping coefficient is

$$\zeta = -\ln \frac{\left(\frac{A_2}{A_1}\right)}{\left(\Pi^2 + \left[\ln\left(\frac{A_2}{A_1}\right)\right]^2\right)^{1/2}} \quad (3)$$

A graphical solution of equation 3 is shown in figure 10.

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